On the Influence of Ion Sheaths upon the Resonance Behaviour of a r. f. Plasma Probe

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The applicability of the r.f. resonance-probe method to the measurement of the electron density in a plasma is investigated in a thermal plasma of low density. The experiments show that in contrast with the investigations published so far, the resonance frequency determined by the r.f. probe does not agree with the plasma frequency but is found always to be lower than this. The difference between resonance frequency and plasma frequency is caused by the ion sheath in front of the probe; the thickness of the sheath determines the amount of the frequency shift. Therefore, the r.f. resonance-probe determination of the electron density is valid only if the geometrical dimensions of the probe environment are taken into account. By the r.f. probe, information on the plasma ion sheath, namely in front of the probe, can be obtained.

The r. f. resonance-probe method was introduced by TAKAYAMA, IKEGAMI and MIYAZAKI [1] for the determination of electron densities in rarefied plasmas. This method has been theoretically investigated [2] and applied for electron density measurements also in the ionosphere [3].

The r. f. resonance probe method consists in superimposing the electrical potential of a LANGMUIR probe with a r. f. voltage of variable frequency, and in recording the dependence of the direct probe current on the frequency of the r. f. voltage. The r. f. voltage causes an additional direct current which is nearly independent of the frequency at sufficiently low frequencies. At higher frequencies, the direct current increases, reaches a peak at a characteristic frequency, herein called "resonance frequency", and then decreases to zero (cf. 4, 6, and 7). According to earlier experimental and theoretical investigations [1–3] this resonance frequency should be equal to the plasma frequency:

$$\omega_p = \sqrt{4\pi \frac{e^2 n_e}{m_e}} = 2\pi \cdot 8.97 \cdot 10^3 \cdot \sqrt{n_e(\text{cm}^{-3})}\ \text{cps} \tag{1}$$

(e and m_e stand for electron charge and mass, respectively). Thus, the resonance frequency "ω_p" would be determined by the electron density n_e. Instead our measurements in a thermal cesium plasma have shown that the resonance frequency is always different from the plasma frequency.

Different authors [4, 5] have already pointed out that the resonance behaviour of a spatially bounded plasma depends on the shape of its boundaries. In applying the r. f. resonance-probe method, the ion sheaths between probe and plasma as well as between plasma and reference electrode have to be included in the consideration of the resonance behaviour.

As a simple model for the interpretation of the resonance behaviour of a bounded plasma, a plane "plasma capacitor" (cf. Fig. 1) can be used. That this is a good approximation has been experimentally verified e. g. by MAYER [4].

For the plasma between the two sheaths (separated by a distance p), a dielectric constant of

$$\varepsilon_p = 1 - \omega_p^2/(\omega^2 - i\,\omega\,\nu_c) \tag{2}$$

(ν_c is the collision frequency) is assumed.

[1] K. TAKAYAMA, H. IKEGAMI, and S. MIYAZAKI, Phys. Rev. Letters **5**, 238 [1960].
[2] Y. H. ICHIKAWA and H. IKEGAMI, Progr. Theor. Phys. **28**, 315 [1962].
[3] S. MIYAZAKI, K. HIRAO, Y. AONO, K. TAKAYAMA, H. IKEGAMI, and T. ICHIKAWA, Rep. of Ionosphere and Space Research in Japan, Vol. XIV, 148 [1960].
[4] H. M. MAYER, 6. Intern. Conf. on Ionization Phenomena in Gases, Paris 1963.
[5] W. O. SCHUMANN, Z. Naturforschg. **13 a**, 888 [1958].

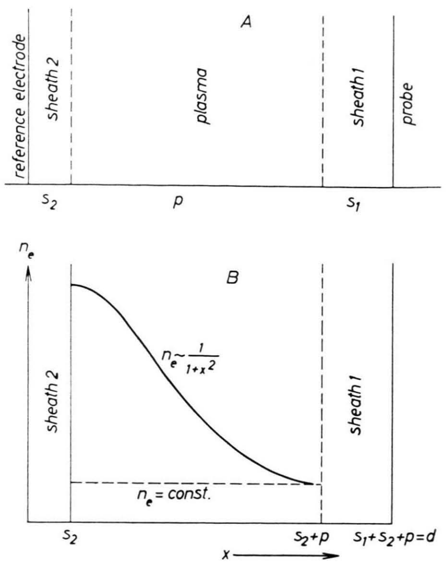

Fig. 1. Model of the plasma condensor (A) and the adopted electron density distribution within it (B).

The sheaths separating the "capacitor plates" from the plasma have thicknesses s_1 (between the probe and the plasma) and s_2 (between the reference electrode and the plasma). The dielectric constant for the sheaths is that for vacuum: $\varepsilon_s = 1$. Simple calculations, already carried out by MAYER [4], result in an "effective" dielectric constant for the plasma capacitor:

$$\varepsilon_{\text{eff}} = 1 - (\omega_{\text{p}}^2 - \omega_{\text{res}}^2) \frac{\omega^2 - \omega_{\text{res}}^2 + i\,\omega\,\nu_{\text{c}}}{(\omega^2 - \omega_{\text{res}}^2)^2 + \omega^2\,\nu_{\text{c}}^2} \tag{3}$$

with a resonance frequency, ω_{res}, which is different from ω_{p},

$$\omega_{\text{res}} = \omega_{\text{p}} \sqrt{(s_1+s_2)/(s_1+p+s_2)} < \omega_{\text{p}}\,. \tag{4}$$

These considerations on the resonance behaviour of the dielectric "constant" ε_{eff} are apparently also valid for the behaviour of the additional direct probe current, which is caused by superimposing a suitable r. f. voltage upon the d. c. probe potential. This has experimentally been shown by PETER et al. [6]. Numerical computations, carried out by WIMMEL [7], also show that the r. f. resonance of the MAYER model [4] provides for a d. c. resonance to occur at $\omega \approx \omega_{\text{res}} < \omega_{\text{p}}$. WIMMEL [7] gives also a critical review of the theory by ICHIKAWA and IKEGAMI [2].

Experiment

The r. f. resonance probe method has been examined in a quiescent thermal Cs-plasma at electron temperatures T_e of about 2000 °K, with a density range of $10^6\ \text{cm}^{-3} \leqq n_e \leqq 5 \cdot 10^8\ \text{cm}^{-3}$, and at a neutral gas pressure p_n lower than 10^{-5} Torr (= mm Hg). The Cs-plasma is generated by contact ionization at a hot tungsten surface. It streams ambipolar into a glass tube, where the plasma density decreases with the distance x from the plasma source as $1/x^2$. The plane r. f. probe is inserted from the other end of the tube and is axially movable so that the distance between probe and plasma emitter can be varied.

First results of our measurements as well as details about the plasma apparatus and measuring technique have already been published [6, 8].

At least at small distances the plane probe together with the plane emitter, which serves as grounded reference electrode in the probe circuit, can be considered as a plane capacitor having an electron density which decreases from the emitter to the probe (cf. Fig. 1 B). In most cases the diameter of the probe is the same as that of the emitter and equals 3 cm. The influence of the variable electron density on the resonance freqency has been calculated (see appendix). The ratio of resonance to plasma frequency in front of the probe sheath has been found in first approximation to be determined by the same geometric function as that given by MAYER [4], though the value of this ratio exceeds by up to 70% that for a plasma with spatially constant electron density. This guarantees that the model of the plane plasma capacitor may also be applied to the experiments with the r. f. resonance probe discussed here.

Experimental Results

a) The electron density measured by the LANGMUIR probe method decreases with the distanxe x of the probe from the plama source (cf. Fig. 2 A) as $1/x^2$.

The simultaneously measured resonance frequencies have been found to be below the corresponding plasma frequencies by a factor independent of x and about equal to 0.3 (cf. Fig. 2 B).

This experimental result can be understood by referring to the plasma capacitor model. If the sheath thickness s_1 in front of the probe is replaced by a multiple, a, of the DEBYE length λ_D (cf. OTT [9])

$$s_1 = a \cdot \lambda_D$$

with (5)

$$\lambda_D = 6.9 \cdot \sqrt{T_e(°\text{K})/n_e(\text{cm}^{-3})}\ \text{cm}$$

[6] G. PETER, G. MÜLLER, and H. H. RABBEN, Proc. 6. Intern. Conf. on Ionization Phenomena in Gases, Paris 1963, 147 [1964].

[7] H. K. WIMMEL, Lab. Report IPP 6/11, Institut für Plasmaphysik, Garching bei München (1963). — Lab. Report IPP 6/16, Institut für Plasmaphysik, Garching bei München (1964). — Z. Naturforschg. **19 a**, 1099 [1964].

[8] G. PETER, G. MÜLLER, and H. H. RABBEN, Lab. Report IPP 2/32, Institut für Plasmaphysik, Garching bei München (1963).

[9] W. OTT, Z. Naturforschg. **17 a**, 962 [1962].

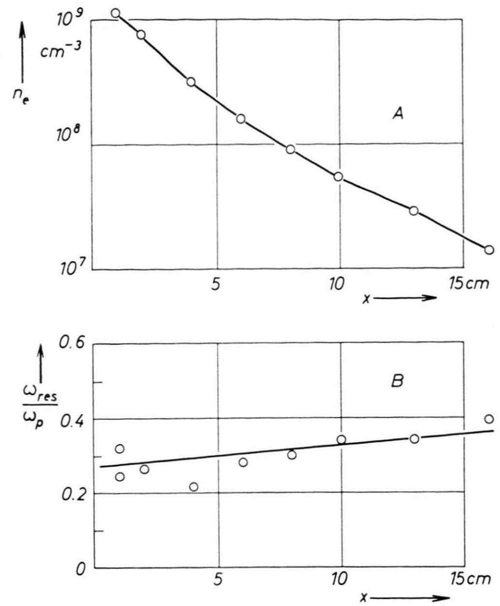

Fig. 2. Electron density (A) and ratio of resonance frequency to plasma frequency (B) as a function of the distance x.

and the density n_e is replaced by (n_0/x^2), (s_2, the sheath thickness in front of the emitter, can be neglected compared with s_1 because of the higher density at the emitter), and if further eq. (5) is substituted for s_1 in eq. (4), then the ratio of the resonance to the plasma frequency is given by

$$\frac{\omega_{\mathrm{res}}}{\omega_{\mathrm{p}}} \propto \sqrt{a \cdot \lambda_{\mathrm{D,E}}} \tag{6}$$

where $\lambda_{\mathrm{D,E}}$ stands for the DEBYE lenght of the plasma at the emitter.

On the other hand, from the eq. (4) and (5), the sheath thickness in front of the probe, s_1, can be estimated to be about eleven times the DEBYE length λ_{D}. This value may be considered as reasonable in view of the relatively high probe bias of -5 volts.

b) In another experiment the dependence of the resonance frequency on the probe bias has been measured. The ratio of the measured resonance frequency to the plasma frequency is plotted in Fig. 3 A. The electron density in this case was $3.1 \cdot 10^7$ cm^{-3}, the electron temperature 2400 °K, and the r. f. amplitude was set at 0.5, 1, or 3 volts. Fig. 3 A shows a large variation of the resonance frequency with the probe potential:

The resonance frequency approaches the plasma frequency as the negative probe potential is increased. The two frequencies could not be made equal however, since at a very high negative probe potential, the resonance phenomenon disappeared. The probe potential at which the resonance disappeared was more negative at higher r. f. amplitudes.

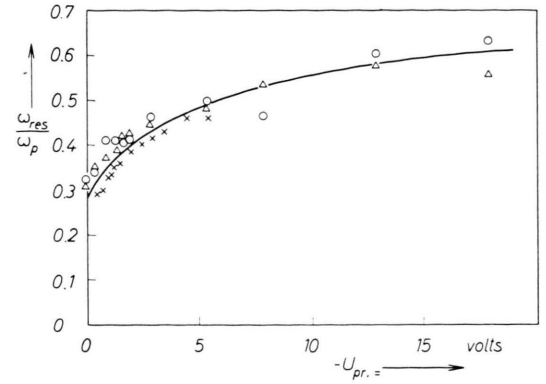

Fig. 3 A. Dependence of the ratio of resonance frequency to plasma frequency on the d. c. probe potential.
× for $U_{\mathrm{HF}}=0.5$ volts, o for $U_{\mathrm{HF}}=1$ volt, Δ for $U_{\mathrm{HF}}=3$ volts.

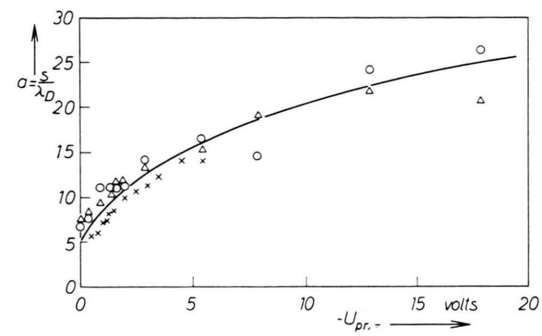

Fig. 3 B. The sheath thickness deduced from the resonance frequency in units of DEBYE lengths.
× for $U_{\mathrm{HF}}=0.5$ volts, o for $U_{\mathrm{HF}}=1$ volt, Δ for $U_{\mathrm{HF}}=3$ volts.

The sheath thickness relative to the DEBYE length has been evaluated from eq. (4) and (5) as described above. It is plotted against the probe potential in Fig. 3 B. The shift of the resonance frequency with the probe potential results from the variation of the sheath thickness. This can also be shown by CHILD's law for a space-charge limited current (cf. OTT [9]). For a negatively biased plane probe which only receives ion current from the plasma CHILD's $U^{3/2}$-law:

$$I_{\mathrm{pr}}^{+} \propto \frac{U_{\mathrm{pr}}^{3/2}}{s_1^2} \tag{7}$$

must be valid. With eq. (6) the relation

$$\frac{\omega_{\mathrm{res}}}{\omega_{\mathrm{p}}} \cdot I_{\mathrm{pr}}^{+1/4} \propto U_{\mathrm{pr}}^{3/8} \tag{8}$$

is obtained. In Fig. 4, the measurements represented in Fig. 3 A are used to plot the logarithm of the left side of eq. (8) against the logarithm of the negative d. c. probe potential. The converted measuring values verify very well the relation (8) and with it the influence of the sheath density s_1 in front of the probe upon the resonance behaviour.

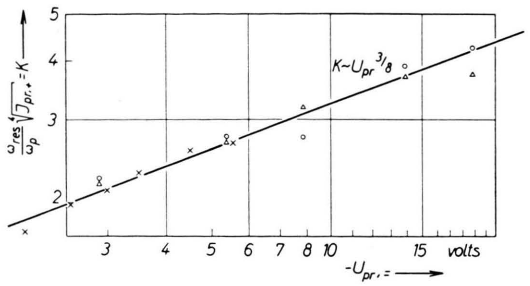

Fig. 4. Experimental verification of the relation of eq. (8). × for $U_{HF}=0.5$ volts, o for $U_{HF}=1$ volt, Δ for $U_{HF}=3$ volts.

c) In a third experiment, with the probe at a given position in the plasma vessel, the electron density has been varied over a large range (from $5 \cdot 10^6$ to $5 \cdot 10^8$ electrons/cm^3) and, with the probe at floating potential, the r. f. probe resonance frequency has been measured. The results are shown by the plot of Fig. 5. The plasma frequency, as deduced from the measured LANGMUIR density, was considerably

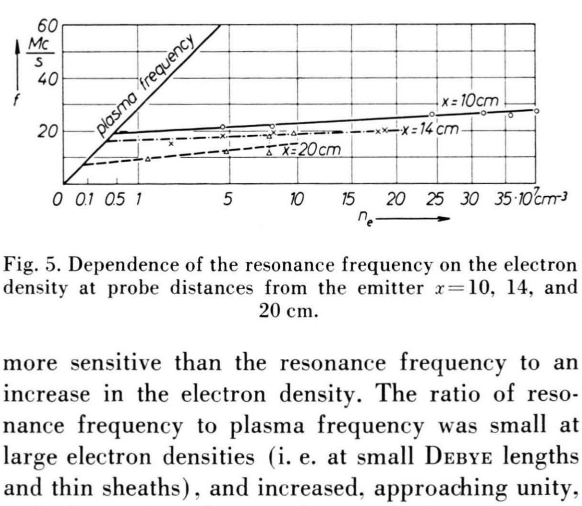

Fig. 5. Dependence of the resonance frequency on the electron density at probe distances from the emitter $x=10$, 14, and 20 cm.

more sensitive than the resonance frequency to an increase in the electron density. The ratio of resonance frequency to plasma frequency was small at large electron densities (i. e. at small DEBYE lengths and thin sheaths), and increased, approaching unity, with decreasing electron density. Before the ratio reached unity, the r. f. probe resonance disappeared. This can be understood from eq. (3): For the case $\omega_{res} = \omega_p$, the effective dielectric constant of the equivalent capacitor, ε_{eff}, becomes unity and, therefore, independent of frequency.

Summary of Results

Electron density measurements made with LANGMUIR probes and those made with the r. f. resonance method have been found not to agree. The resonance frequency of the r. f. probe is always lower than the plasma frequency. By several experiments, it has been confirmed that the resonance behaviour of a bounded plasma, and in particular the measured resonance frequency of a r. f. probe, are essentially determined by the plasma-bounding ion sheaths. The dependence of the resonance frequency of the r. f. probe on the sheath thickness is investigated by varying the d. c. probe potential as well as the electron density of the plasma. The increase of the sheath thickness in front of the probe can be shown to determine the deviation of the resonance frequency from the plasma frequency. Therefore, the resonance frequency results not only form the electron density of the plasma but also from the sheath thickness according to eq. (4). The r. f. resonance probe method can consequently be used for precise electron density determination only as long as the sheath conditions of the plasma are known. On the other hand the r. f. pobe method gives information on the plasma sheaths if the plasma properties are known.

In addition, our experiments permit conclusions on the reliability of the density measurements with LANGMUIR probes. Our density measurements by the LANGMUIR method and our measured resonance frequencies are consistent with MAYER's model [4]. Hence it can be concluded, that it is possible to determine the electron density from the intersection point of the extrapolated deceleration and saturation sections of the electron courrent curve in the semilogarithmical LANGMUIR V-A-characteristic, and thus obtain a reliable value for the electron density of the plasma.

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Appendix

Equation (3) has been derived by MAYER [4] for a homogeneous plasma between two plane probes with ion sheaths. With the apparatus used here, however, the electron density decreases from the emitter to the probe by up to two orders of magnitude. Therefore, the applicability of eq. (3) to such an inhomogeneous plasma has to be verified. For this purpose, the effective dielectric constant

$$\varepsilon_{\mathrm{eff}} = d \Big/ \Big(\int_0^{d-s} (1/\varepsilon_{\mathrm{p}})\,\mathrm{d}x + \int_{d-s}^{d} \mathrm{d}x \Big)$$

with d = distance between emitter and probe, s = thickness of the sheath in front of the probe and $\varepsilon_{\mathrm{p}} = 1 - \omega_{\mathrm{p}}^2/\omega^2$, has been calculated for a system consisting of a plasma column with length $p = d - s$ and electron density decreasing with the distance from emitter as $1/(1+x^2)$ (cf. Fig. 1 B).

From this, the resonance frequency ω_{res} has been derived, i. e. the frequency at which the dielectric constant $\varepsilon_{\mathrm{eff}}$ has a maximum. The result of the numerical computation is plotted in Fig. 6. As in the case of a homogeneous plasma, the resonance frequency is shown always to be smaller than the plasma frequency, due to the electron density at the sheath boundary in front of the probe. The dependence of the resonance frequency on the relative sheath thickness s/d is also similar to that for a homogeneous plasma, except that at small d the value of the resonance frequency exceeds by up to 70% that of the resonance frequency for a plasma with spatially constant electron density.

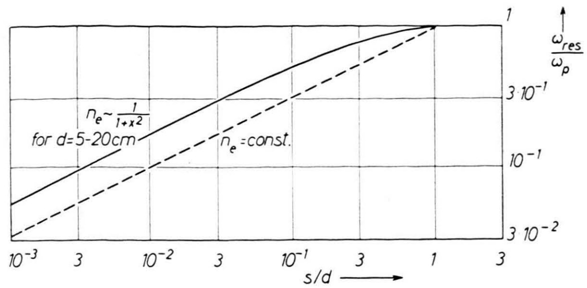

Fig. 6. Dependence of resonance frequency ω_{res} on the sheath thickness s/d for a homogeneous plasma (dashed curve) and for an inhomogeneous plasma with a density dependence proportional to $1/(1+x^2)$ (full curve).

Magnetfeldmessungen mit Hilfe des Zeeman-Effektes*

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A method for the measurement of time-variable magnetic fields from the Zeeman effect of time-variable line emission is described. This method is independent of the special line profile and of time variations in intensity, line profile and line shift. The sensitivity of the method and the accuracy of the measurement are discussed. Test measurements were made in a nitrogen discharge with an external homogeneous magnetic field having 130 kc/sec frequency. — The measurement of inhomogeneous fields from spatial inhomogeneous line emission gives a mean value of the field which depends on the spatial distribution of the line emission. Nevertheless, measurements made on a little fast Θ-pinch with a N II line, which was emitted from the hot plasma, were reproduceable.

1. Zusammenstellung bisher angewendeter Verfahren

Die magnetische Beeinflussung der Spektrallinien gestattet es, Magnetfelder auf optischem Wege zu messen. Wegen des Aufwandes wird von dieser Möglichkeit nur dann Gebrauch gemacht, wenn direkte Methoden versagen.

In der Astrophysik wurde ein Verfahren entwickelt, die zeitlich nahezu konstanten Sonnenmagnetfelder aus dem Zeeman-Effekt der Absorptionslinien zu bestimmen. Die Linienbreiten betragen dort das 10^2- bis 10^3-fache der Aufspaltungen. Um die Komponenten trotzdem trennen zu können, wird nach Hale [1] der longitudinale normale Zeeman-Effekt benutzt, bei dem statt einer Linie zwei erscheinen, die entgegengesetzt zirkular polarisiert sind und um den Betrag

$$\Delta\lambda = 4{,}66 \cdot 10^{-13}\, \lambda_0^2\, B \quad (\Delta\lambda,\ \lambda_0 \text{ in Å},\ B \text{ in Gauß}) \tag{1}$$

nach größeren und kleineren Wellenlängen verschoben sind. λ_0 ist die Wellenlänge der feldfreien Linienmitte, B ist das Magnetfeld. Durch eine $\lambda/4$-Platte werden die entgegengesetzt zirkular polari-

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[1] G. E. Hale, Astrophys. J. **38**, 27 [1913].